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J. Phys. A: Math. Gen. 19 (1986) L693-L698. Printed in Great Britain

LETTER TO THE EDITOR

Diffusion on percolation clusters with a bias in topological space: non-universal behaviour

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Received 12 February 1986

Abstract. We study diffusion on the infinite percolation cluster above the percolation threshold, $p > p_c$, under the influence of a constant bias field E in topological space ('topological bias'). We find that above a critical bias field $E_c(p)$ diffusion is anomalous and non-universal: the diffusion exponent d_w^l increases with E as $d_w^l = A(p)|\ln[(1-E)/(1+E)]|$, while $A(p)$ decreases monotonically with concentration p. This intrinsic anomalous behaviour is supported in a wide range of concentrations $p > p_c$ by extensive numerical simulations using the exact enumeration method.

In recent years, the problem of diffusion in random structures has been studied extensively. While some progress has been made in understanding non-biased diffusion in random media [1-7], including anomalous diffusion at the critical concentration p_c in percolation systems, diffusion under the influence of a bias field so far has been mostly controversially discussed [8-17]. Even in the comparatively simple and well defined percolation system the overall behaviour has not yet been understood.

In this letter we study the effect of a bias field in topological ('chemical') space on the diffusion properties in percolation systems [18] *above* the critical concentration, $p > p_c$. In the topological bias, a random walker is pushed to 'move away' in chemical distance (or path length) from the origin of the force (see figure 1). This model is relevant, e.g., to the problem of flow of compressible fluids in random media where a pressure source is applied at one point of the system. In this case the pressure source gives rise to a topological bias rather than a Pythagorean bias. This concept of a bias in topological space has been introduced recently by Bunde *et al* [19] and was applied to a simple model of random media, which has been topologically mapped on a random comb with an exponential distribution of the length of the teeth. The problem is controlled by those teeth (dangling ends) in which particles spend an extremely long time. The mean path length $\langle l \rangle$ a random walker travels in time t can be written as [6]

$$\langle l \rangle \sim t^{1/d_w^l} \qquad (1)$$

where d_w^l is the diffusion exponent. It has been found [19] that above a bias threshold E_c, diffusion is anomalous and non-universal in two respects: the diffusion exponent d_w^l is above 1 and increases continuously with the magnitude E of the bias. In addition, d_w^l increases monotonically with the mean tooth length. It has been argued that these features are characteristic for random media with topological bias. Here we present

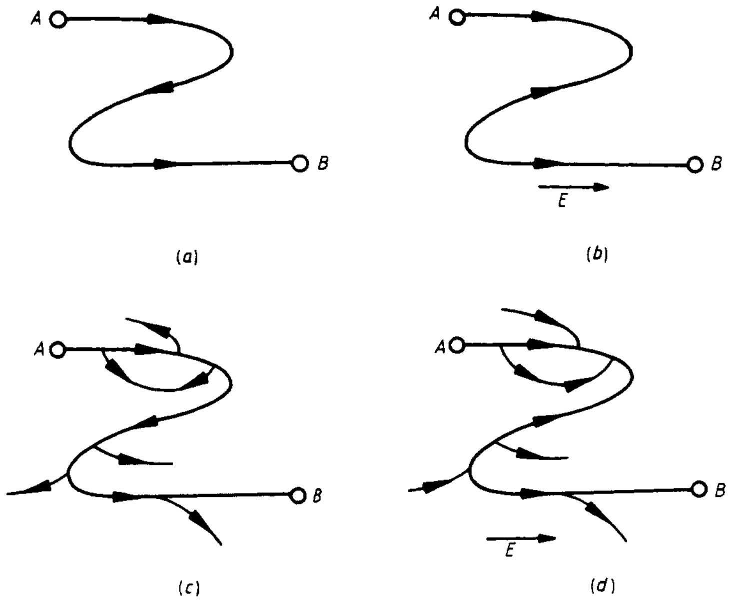

Figure 1. Illustration of the differences between topological bias and Pythagorean bias. The arrows represent the direction of the bias field for the topological bias, (a) and (c), and for the Pythagorean bias, (b) and (d). In (a) and (b) are shown the directions of the field along the shortest path between A and B; in (c) and (d), loops and dangling ends are also included. Note that for the topological bias every loop and dangling end acts as a random delay, in contrast to the situation with a Pythagorean bias.

analytical and numerical evidence that the anomalous and non-universal behaviour of the diffusion exponent indeed applies to percolation systems above the critical concentration. In contrast to non-biased diffusion which shows anomalous long-time behaviour *only at criticality*, $p = p_c$, we find that topologically biased diffusion shows anomalous behaviour *also above* p_c; the diffusion exponent changes continuously with the field and the concentration.

In our model, the topological bias is defined as follows: a random walker has an enhanced probability $p_+ \sim (1+E)$ to increase the chemical distance (path length) from the source in the next step, and the decreased probability $p_- \sim (1-E)$ to decrease the chemical distance (see figure 1).

Above the critical concentration the infinite cluster consists of a backbone and of dangling ends emanating from the backbone. In the absence of dangling ends and loops, we expect normal biased behaviour, $\langle l \rangle \sim t$. The effect of dangling ends and loops is to provide random relays (waiting times) along the backbone, since the topological bias drives the walker toward the tips of the dangling ends (see figure 1). Thus both dangling ends and loops in the backbone act as effective dead ends. The waiting time is the characteristic time the walker spends on a dead end. For a simple dead end (without loops and branches) of length L the waiting time τ scales as ([12], see also [19])

$$\tau \sim \left(\frac{1+E}{1-E}\right)^L . \qquad (2)$$

We assume now that the effect of branching and loops in the dangling ends as well as longer paths on the backbone can be taken into account by introducing an effective dead end length L_e which replaces L in (2). Furthermore we assume that the effective lengths L_e have the exponential distribution

$$P_0(L_e) \sim \exp[-L_e/\langle L_e \rangle] \tag{3}$$

where $\langle L_e \rangle$ is the mean effective length of the dead ends. From (2) and (3) we find that the waiting times τ satisfy the distribution

$$P(\tau) \sim \tau^{-\gamma} \tag{4a}$$

where

$$\gamma = 1 + \{\langle L_e \rangle \ln[(1+E)/(1-E)]\}^{-1}. \tag{4b}$$

Since asymptotically most of the time is spent in the effective dead ends, the time t_{AB} the walker needs to move between two points A and B under the influence of the bias field scales as [19]

$$t_{AB} \sim \sum_{i=1}^{N_{AB}} \tau_i. \tag{5}$$

Here the sum runs over all the N_{AB} dangling ends which emanate from the backbone between A and B. Following similar scaling arguments as in [19] and [21] we find for $\gamma < 2$ (see also [20])

$$t_{AB} \sim l_{AB} \int_1^{\tau_{\mathrm{MAX}}} \tau P(\tau)\, \mathrm{d}\tau \sim l_{AB} \tau_{\mathrm{MAX}}^{2-\gamma} \tag{6}$$

where l_{AB} is the length of the shortest path between A and B. For $\gamma > 2$ we obtain $t_{AB} \sim l_{AB}$, the result for Euclidean lattices. Since $\tau_{\mathrm{MAX}} \sim l_{AB}^{1/(\gamma-1)}$ [19] we obtain $t_{AB} \sim l_{AB}^{d_w^l}$ with

$$d_w^l = \begin{cases} 1 & E \leqslant E_c \\ \langle L_e \rangle \ln[(1+E)/(1-E)] & E > E_c \end{cases} \tag{7a}$$

where

$$E_c = \tanh\left(\frac{1}{2\langle L_e \rangle}\right). \tag{7b}$$

By definition, $\langle L_e \rangle \to 0$ for $p \to 1$. We expect that $\langle L_e \rangle$ increases monotonically for $p \to p_c$ from above and diverges at p_c. Hence $E_c \to 0$ for $p \to p_c$ and $E_c \to 1$ for $p \to 1$.

To test our prediction (7*a*, *b*) we have performed extensive computer simulations of topologically biased diffusion on the 2D infinite percolation cluster above p_c for several values of concentration p and bias field E. We have used the exact enumeration method [3, 22] for calculating $\langle l \rangle$ as a function of time from which we deduced d_w^l. Our results are for the square lattice where $p_c \simeq 0.593$. For each set of p and E we averaged over 100 clusters of 250 shells each and calculated $\langle l(t) \rangle$ for 5×10^3 time steps; the results for the concentrations $p = 0.623$, 0.653 and 0.703 are shown in figure 2. In figure 3 we have plotted d_w^l, obtained from the asymptotic slopes of figure 2, as a function of $\ln[(1+E)/(1-E)]$. The linear curves we obtain strongly support our results (equation (7*a*)). Note that the extrapolations of all lines cross the origin, in agreement with (7).

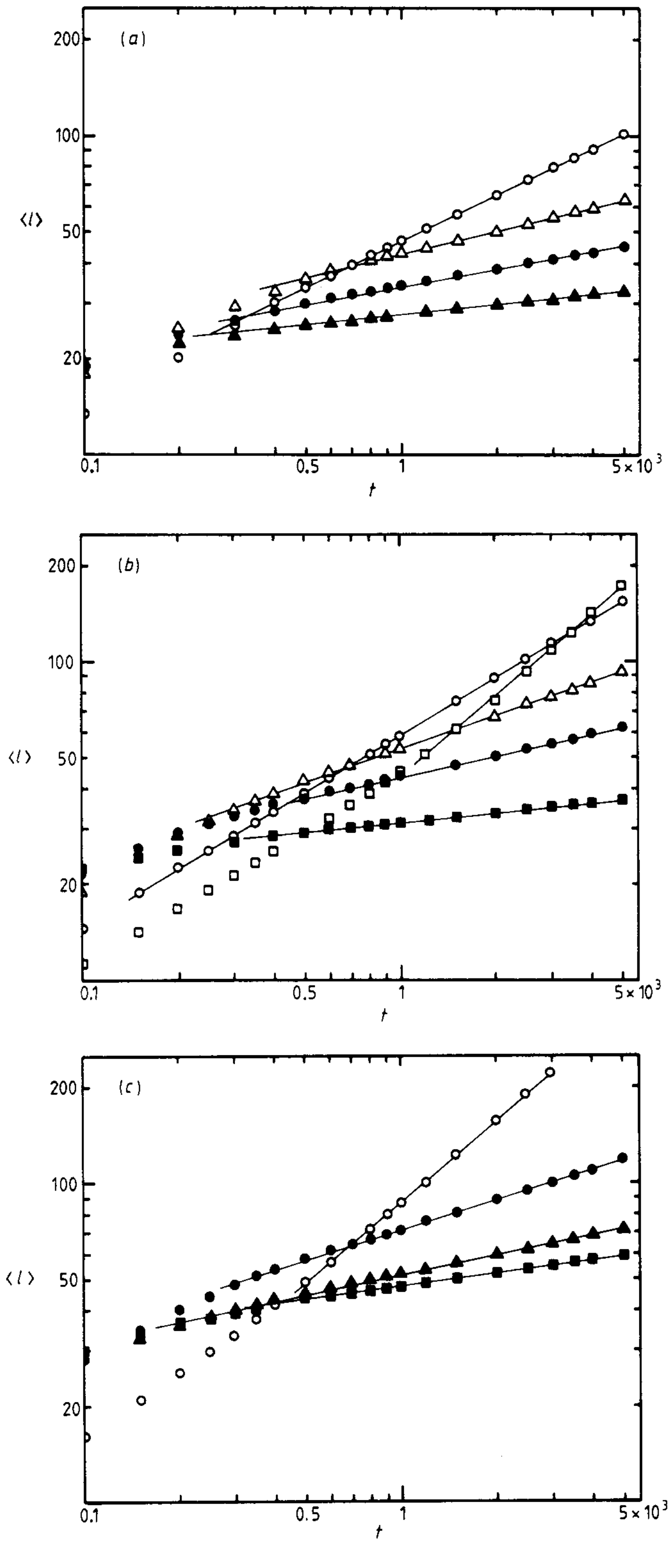

Figure 2. Plot of the mean path length $\langle l \rangle$ travelled by a random walker on the infinite percolation cluster above p_c under the influence of the topological bias field E, as a function of time t. The starting point of the walkers was the origin of the bias field. The results are for $E = 0.1$ (□), 0.2 (○), 0.4 (△), 0.6 (●), 0.8 (▲) and 0.9 (■) and for $p = 0.623$ (a), $p = 0.653$ (b) and $p = 0.703$ (c).

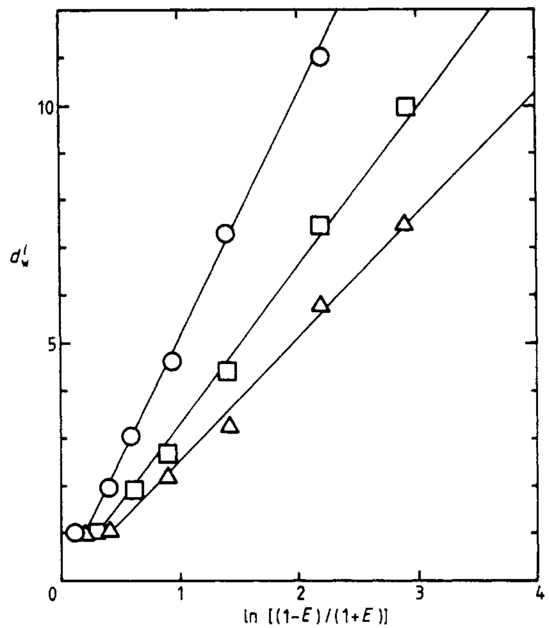

Figure 3. The diffusion exponent d_w^l as a function of $\ln[(1+E)/(1-E)]$ for several values of p: $p = 0.623$ (○), $p = 0.653$ (□) and $p = 0.703$ (△).

Our results (7*a*, *b*) crucially depend on the assumption that the distribution of L_e has a simple exponential form, (3). A more general form is $P_0(L_e) \sim \exp[-L_e/\langle L_e\rangle)^{1-\delta}]$ with δ less than one. However we can argue that for $\delta < 0$ one obtains always $\langle l\rangle \sim t$ for large times, i.e. $d_w^l = 1$. This case can be clearly excluded from our numerical data (figure 2). Our data strongly support the case $\delta = 0$ where a dynamical phase transition occurs from $d_w^l = 1$ to $d_w^l > 1$. However due to the error bars in our data which are about 5%, we cannot exclude the case $0 < \delta \ll 1$. In this case the curves look similar to those for $\delta = 0$, but the transition is smeared out. In order to exclude $\delta = 0$ the accuracy of the data must be enhanced significantly. Since in our study about 10 h CPU time of an IBM 3081 computer were needed for each set of p and E, it is not likely that this problem can be easily solved.

In summary, we have discussed the effect of a topological bias on the diffusion of particles on percolation clusters above p_c and found theoretical and numerical evidence for a dynamical phase transition. For every concentration $p > p_c$ there exists a critical bias field E_c. For $E < E_c$, the diffusion exponent remains at its classical value, $d_w^l = 1$, while for $E > E_c$, d_w^l increases monotonically with E. In contrast it was found recently that at $p = p_c$ there exists no dynamical phase transition and $\langle l\rangle \sim A \ln t$; the amplitude A depends on E [23].

It is interesting to compare these findings with previous results for the more common Pythagorean bias [8-17]. Above p_c, the existence of a critical Pythagorean bias field has been suggested by Barma and Dhar [10] and was questioned by Gefen and Goldhirsch [16] as well as by Pandey [13]. Also, the question as to whether the diffusion exponent could vary with the Pythagorean field has been discussed by Dhar [11]. Very recently, numerical evidence for a $\ln t$ behaviour of the mean displacement at p_c has been given by Stauffer [17], which is in qualitative agreement with the analytical and numerical findings of Havlin *et al* [23] for the topological bias. We believe that in general the effects of the topological and Pythagorean bias on the diffusion are similar. However, they might be easier to detect numerically for the topological bias. In contrast to the Pythagorean bias, in the topological bias *all* dead ends delay the diffusion and therefore the effects of the field are more pronounced.

We thank D Ben-Avraham, H Sompolinsky, I Webman and G H Weiss for valuable discussions; in particular we are indebted to D Stauffer for valuable suggestions. This work has been supported by the USA-Israel Bi-National Foundation, by Minerva, by Deutsche Forschungsgemeinschaft and by grants from ONR and NSF.

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